Journal of Process Control 23 (2013) 1063–1074

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Dimensional analysis approach to dominant three-pole placement in delayed PID control loops

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ARTICLE INFO

Article history:
Received 29 November 2012
Received in revised form 4 June 2013
Accepted 5 June 2013
Available online 20 July 2013

Keywords:
Dimensional analysis
Dominant pole placement
PID controller tuning
Ultimate frequency
Dominance index

ABSTRACT

PID control loops with time delay are characterized by infinite number of poles but the pole assignment technique for adjusting the controller parameters can be applied to placing *three poles* only. The dominance of these poles is therefore an essential condition for this application. A novel approach to this problem involves applying *dimensional analysis* theory to obtain a generalized model of the control loop and then to perform a parameter tuning for its *dimensionless representation.* A one-row dimensional matrix results from the assumption of the usual dimensionless interpretation of both control error and actuating signals of the controller. Dimensionless similarity numbers of the so-called *swingability* and *laggardness* are introduced to specify the plant dynamics in the controller synthesis. A trio of numbers is assigned to become the dominant zeros of the characteristic quasi-polynomial of the control loop and the corresponding PID parameter adjustment is derived in the form of uniform formulae. The tuning of the proper damping and the real pole position ratios is provided by means of an IAE optimization technique. A *dominance degree* notion is introduced and an argument increment criterion is proposed to check the dominance of any of the pole placement cases. The quality of the disturbance rejection response is taken as the general criterion in the design of the time delay plant control.

1. Introduction

Pole assignment is a widely used approach to the state space system design that is also applied in methods for tuning the controllers. For tuning three PID parameters just *three poles* of the control loop are suited to be assigned and with regard to this any assumptions of rather sophisticated or higher order process models turn out to be inadequate. On the other hand, as a matter of fact, the time delay effect can be viewed as a common process property which, however, leads to *infinite spectrum* of control loop poles and to a need to investigate their *dominance* as a crucial issue in the pole placement.

The dependence of the dominant pair of closed-loop poles on the controller parameters was first investigated by Hwang and Chang [1] by means of the Taylor expansion about the critical gain. Instead of *dominant* the term "leading poles" was used in this paper. Dominant pole placement design was introduced somewhat differently by Persson and Åström [2] and was further explained in Åström and Hägglund [3]. At about the same time Hwang and Fang [4] published an extensive optimization study on dominant pole placement for first and second order time delay plants. Numerous methods with modified specifications of tuning conditions were presented subsequently and a survey was made by O'Dwyer [5].

Applying the pole assignment approach to systems with delay (i.e. with infinite spectrum of poles) has led to the specific problem of how to select a proper prescription of the pole positions in order to obtain capable candidates of dominant poles that really do determine the behaviour of the system. Any pole placement in a time delay system is always connected with the risk that although the prescribed poles are achieved in the system spectrum they may lose any meaning because some *other poles* spontaneously take over the dominant position in the infinite system spectrum. Consequently any result of a pole assignment of this kind can be approved as valid only after checking that the placed poles really have reached the dominant positions. To the best of the authors' knowledge no general theorem is yet available that guarantees in advance that a chosen prescription of poles for a time delay system will reliably result just in the group of system dominant poles.

The pole placement approach in a control loop with delay is to be considered only for placing a small group of *dominant* poles – either a complex conjugate pair or a three-pole group, usually one pair with a real pole. The key issue of selecting the prescribed poles in a way that guarantees their dominant position was investigated by Wang et al. in [6]. Because of the number of three controller parameters only the three-pole option $p_{1,2,3}$ can reasonably be prescribed in assigning the poles for tuning the PID controller for a time

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http://dx.doi.org/10.1016/j.jprocont.2013.06.001

delay plant. However, in a considerable number of papers the dominant pole placement in PID tuning has also been considered for a *single pair* of complex conjugate poles $p_{1,2} = -\alpha \pm j\Omega, \alpha > 0$, assigned to take up the rightmost position in the system spectrum and satisfying an additional requirement of a specified frequency response for control synthesis. Various combined strategies have been presented, e.g. in [7,8] or [9]. The case of placing three prescribed poles, a complex conjugate pair $p_{1,2}$ and a real pole $p_3 = -\beta$ as dominant was solved by Hwang and Fang [4]. A guarantee of dominance in the pole placement based on the root locus and Nyquist plot applications was presented by Wang et al. [6]. The dominant pole placement may also be performed in an iterative way, as a series of attempts that shift the prescribed poles to the left as in [10], or a modified optimization in [11].

The rest of the paper is structured as follows. The dimensionless description of the control loop is introduced in Section 2 and an ultimate angle as similarity number for the critical control setting is presented in Section 3. Section 4 presents explicit formulae for setting the dimensionless control parameters for various types of plants, and Sections 5 and 6 deal with the issue of the pole dominance, in the former by means of an argument increment condition and in the latter via an IAE optimization technique. Sections 7 and 8 present an application example and concluding remarks respectively. A brief appendix is added concerning the issue of dimensionless model identification.

2. Control loop dimensionless representation

Although *dimensional analysis* originated and is typically applied in other fields of science it also has a potential in investigations of the control system dynamics [12]. So far the benefit of the dimensional approach to control system design has been limited to specific control areas, where various dimensions of controlled and actuating variables can be considered, as e.g. in the vehicle control [13]. Our aim is to investigate the relationships between the PID controller settings and the control loop dynamics *in general*. As to the dimensional analysis technique presented here we follow the monograph [14] but at the same time we revere the concept commonly accepted in the control science that the particular physical variables of both the plant output and input are represented in controller operation by signals expressed as *percentage* of the control instrument's range, i.e. commonly conceived as *dimensionless variables*. Due to this established concept of control theory a consideration of physical dimensions in terms of mass, length, force etc. turns out redundant and nothing but time remains from the basic dimensional SI units for dimensional investigations in control loop dynamics. The fact that nothing but time and frequency dimensions and their powers may appear in the *dimensional set* of any control problem as soon as we accept the assumption of dimensionless controller input and output significantly simplifies the dimensional analysis considerations. The key concept in applying the dimensional analysis consists in selecting the rules of *dimensional similarity* and in formulating the relationships between the derived *dimensionless variables* as it follows from the general well-known Buckingham theorem.

In investigating the dependence of the dominant closed loop poles on the PI or PID controller parameters it has already been validated that higher order plant models are not necessary for ultimate frequency based methods of controller tuning [1]. In the present work we consider a linear plant described by a second order differential equation with an input delay

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0 y(t) = K a_0 u(t-\tau), \quad a_1 > 0, \quad a_0 > 0, \quad K \neq 0, \quad \tau > 0 \tag{1}$$

able to express the dynamics of a rather wide class of stable processes free of the non-minimum phase zero effect. The special case of the integrating plant with $a_0 = 0$ will be introduced later. The considered control loop is completed by the ideal PID controller described by the equation

$$\frac{du}{dt} = r_0\frac{de}{dt} + r_D\frac{d^2e}{dt^2} + r_I e \tag{2}$$

where e is the control error, $e = w - y$. As noted above, if y, u, w and e are considered as dimensionless the coefficients in (1) and (2) are of dimensions given only as time powers. These five coefficients primarily belong into the dimensional set of the control loop, namely: $r_D[\mathrm{s}]$, $r_I[\mathrm{s}^{-1}]$, $a_1[\mathrm{s}^{-1}]$, $a_0[\mathrm{s}^{-2}]$, $\tau[\mathrm{s}]$, Dimensionless parameters K and r_0 need not necessarily be included into the dimensional set. On the contrary, due to the derivatives in (1) and (2) time t is dimensionally significant variable in the dynamic considerations and as a decisive dynamic parameter of the control loop the ultimate frequency $\omega_k[\mathrm{s}^{-1}]$ of plant (1) (at which the relay feedback control loop oscillates) is to be considered in the relevant dimensional set. Then the following one-row dimensional matrix with seven relevant variables ($n_V = 7$) and one basic dimension [s] of SI ($n_D = 1$) is obtained

$$\begin{array}{ccccccccc} & \omega_k & r_D & r_I & a_1 & a_0 & t & \tau & \\ \mathbf{D} = & [-1, & 1, & -1, & -1, & -2 & 1 & 1] & [\mathrm{s}] \end{array} \tag{3}$$

Unlike the usual dimensional matrices, e.g. in hydrodynamics or thermodynamics the usual rows for length, mass or temperature are missing here. For the integrating plant with $a_0 = 0$ we set b_0 instead of a_0 into $\mathbf{D}$. As to the *order* of variables in $\mathbf{D}$, ω_k as the dependent one, is in the leftmost position and the delay as the crucial factor in the controller setting is in the rightmost position. The set of six dimensionless variables corresponding to this dimensional set results from the following Lemma.

Lemma 1 (.). *Consider a PID control loop composed of (1) and (2) with the relevant dimensional set given by matrix (3). Then there are six mutually independent dimensionless parameters π_i $i = 1, 2, \ldots, 6$ in case of $a_0 > 0$ given by the following relationships to the original variables*

$$\pi_1 = \omega_k\tau, \quad \pi_2 = \frac{r_D}{\tau}, \quad \pi_3 = r_I\cdot\tau, \quad \pi_4 = a_1\cdot\tau, \quad \pi_5 = a_0\cdot\tau^2, \quad \pi_6 = \frac{t}{\tau} \tag{4}$$

while for the case $a_0 = 0$ the fifth parameter is substituted by $\pi_{5I} = b_0 \cdot \tau^2$.

Proof. The one-row type of $\mathbf{D}$ considerably simplifies the search for the dimensionless arguments expected in the generic form

$$\pi_i = \omega_k^{\varepsilon_1} r_D^{\varepsilon_2} r_I^{\varepsilon_3} a_1^{\varepsilon_4} a_0^{\varepsilon_5} t^{\varepsilon_6} \tau^{\varepsilon_7} \tag{5}$$

Due to the only one row in $\mathbf{D}$ ($n_D = 1$ and rank$[\mathbf{D}] = 1$), the expected number of dimensionless arguments is relatively high, $n_\pi = n_V - n_D = 7-1 = 6$. As in [14] these arguments are found using the augmented matrix equation

$$\begin{bmatrix} \mathbf{I}, & \mathbf{0} \\ \mathbf{B}, & \mathbf{A} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_7 \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbf{I}(6\times 6), \quad \mathbf{0}(6\times 1), \quad \mathbf{B}(1\times 6) \tag{6}$$

where $[\mathbf{B}, \mathbf{A}] = \mathbf{D}$. In fact, the set (6) represents only one linear equation for seven unknowns and hence it allows six solutions independent of each other. To facilitate the procedure five exponents of seven may always be fixed to zero in each of these options and in this manner the following six pairs of nonzero exponents are obtained. Inserting these exponents into (5) the following dimensionless parameters result

$$\begin{aligned}
&1.\ \text{For}\ \ \varepsilon_1 = 1\ \ \text{and}\ \ \varepsilon_{2,3,4,5,6} = 0 \to \varepsilon_7 = 1 \to \pi_1 = \omega_k \tau \\
&2.\ \text{For}\ \ \varepsilon_2 = 1\ \ \text{and}\ \ \varepsilon_{1,3,4,5,6} = 0 \to \varepsilon_7 = -1 \to \pi_2 = \frac{r_D}{\tau} \\
&3.\ \text{For}\ \ \varepsilon_3 = 1\ \ \text{and}\ \ \varepsilon_{1,2,4,5,6} = 0 \to \varepsilon_7 = 1 \to \pi_3 = r_I \cdot \tau \\
&4.\ \text{For}\ \ \varepsilon_4 = 1\ \ \text{and}\ \ \varepsilon_{1,2,3,5,6} = 0 \to \varepsilon_7 = 1 \to \pi_4 = a_1 \cdot \tau \\
&5.\ \text{For}\ \ \varepsilon_5 = 1\ \ \text{and}\ \ \varepsilon_{1,2,3,4,6} = 0 \to \varepsilon_7 = 1 \to \pi_5 = a_0 \cdot \tau^2 \\
&6.\ \text{For}\ \ \varepsilon_6 = 1\ \ \text{and}\ \ \varepsilon_{1,2,3,4,5} = 0 \to \varepsilon_7 = -1 \to \pi_6 = t/\tau
\end{aligned} \tag{7}$$

equal to those in (4). Apparently π_4, π_5 belong to the plant, π_2, π_3 to the controller, while π_6 and π_1 take over the roles of time and of the ultimate frequency respectively. The substitution of π_5 by π_{5I} in the case $a_0 = 0$ is obvious. Of course, the obtained introduction of $\pi_i, i = 1, 2, \ldots, 6$ is partly dependent on the chosen variable order in **D**, particularly alternating the rightmost element in **D** would lead to various sets of dimensionless parameters.■

The prime importance of the obtained dimensionless parameters consists in defining the *dimensional similarity* according to the Buckingham's theorem. As to the plant, for instance, if we substitute π_4, π_5 and $\pi_6 = \bar{t}$ into (1) instead of a_0, a_1, τ and t, the following dimensionless equation is obtained if $a_0 \neq 0$

$$\frac{d^2y(\bar{t})}{d\bar{t}^2} + \pi_4 \frac{dy(\bar{t})}{d\bar{t}} + \pi_5 y(\bar{t}) = K\pi_5 u(\bar{t} - 1), \qquad \pi_4 > 0, \quad \pi_5 > 0, \quad K \neq 0 \tag{8}$$

and the following Corollary holds.

Corollary 1. *Consider a pair of plants, A and B as in (1) where $a_0 \neq 0$ with parameters a_{1A}, a_{0A}, τ_A and a_{1B}, a_{0B}, τ_B satisfying the conditions*

$$\pi_{4A} = a_{1A}\tau_A = \pi_{4B} = a_{1B}\tau_B, \quad \pi_{4A} = a_{0A}\tau_A^2 = \pi_{4B} = a_{0B}\tau_B^2, \quad K_A = K_B, \tag{9}$$

with the same values of π_4, π_5 and K for both A and B. Then both these plants are described by an *identical dimensionless model* (8), common for them, which leads to the same solution $y(\bar{t})$ for both A and B. and due to this property these plants are referred to as *dimensionally similar*.

Proof. Since the statement is a direct consequence of the Buckingham's theorem [14] the *proof* is omitted.■

For example, if the plants A and B of the form (1) have the parameters $a_{1A} = 0.7$, $a_{0A} = 0.1$, $K_A = 2$, $\tau_A = 4$ and $a_{1B} = 3.5$, $a_{0,B} = 2.5$, $K_B = 2$, $\tau_B = 4$, respectively their dimensionless parameters are of the same values $K = 2$, $\pi_4 = 2.8$, $\pi_5 = 1.6$. These plants are *dimensionally similar* because they are described by the same common equation (8) common for both of them. For example their step responses are therefore identical curves if they are plotted in the time scale of $\bar{t}$, i.e. $\bar{t} = t/4$ for A and $\bar{t} = t/0.8$ for B. Obviously yet infinitely many plants like (1) may become similar with A and B if their dimensionless parameters are also equal to $K = 2$, $\pi_4 = 2.8$, $\pi_5 = 1.6$.

Fusion and final form of dimensionless parameters. Although the plant dimensionless parameters π_4 and π_5 are correct as to the theory of dimensional analysis their drawback in describing the similar plants is that both of them primarily indicate rather the delay of the plant response while none of them indicates the other plant property – the character of system (1) *poles*. For a better distinguishing this property of the plant let the following fusions of π_4 and π_5 be introduced

$$\lambda = \frac{\pi_5}{\pi_4^2} = \frac{a_0}{a_1^2} \quad \text{for} \quad a_0 > 0 \tag{10}$$

Parameter λ is suitable to discriminate the nature of the plant as to its poles. It indicates a plant with two real poles if $0 < \lambda \leq 1/4$ and a case of complex conjugate poles if $\lambda > 1/4$, when $a_0 > 0$. The case of $a_0 = 0$ (i.e. λ_I instead of λ) belongs to the integrating plant when one pole is zero. Due to the relationship of λ to possible natural oscillations it will be referred to as the *swingability number* of the plant. On the other hand parameter π_5 indicates to which extent the plant response is lagged and therefore $\pi 4 = \vartheta$ will be referred to as the *laggardness number* and the dimensionless plant equation will then be further considered in the form

$$\frac{d^2y(\bar{t})}{d\bar{t}^2} + \vartheta \frac{dy(\bar{t})}{d\bar{t}} + \vartheta^2 \lambda y(\bar{t}) = K\vartheta^2 \lambda u(\bar{t} - 1), \qquad \vartheta > 0, \quad \lambda > 0, \quad K \neq 0 \tag{11}$$

Furthermore, in investigating the control loop dynamics it is not necessary to distinguish between the gain K of the plant and the controller gains. Therefore K may be fused together with r_0, π_2, π_3 to result in the loop gains for proportional, derivative and integrating actions respectively,

$$r_0 K = \rho_0, \quad \pi_3 K = \rho_I, \quad \pi_2 K = \rho_D \tag{12}$$

A specific meaning has the dimensionless parameter $\pi_1 = \omega_k \tau$. It represents an angle appropriate to both the ultimate frequency and the delay of the plant, therefore it will be referred to as *ultimate angle* Φ_k, $\pi_1 = \omega_k \tau = \Phi_k$. The original set (4) of six dimensionless parameters (or similarity numbers) applicable for the plants with $a_0 > 0$ is then defined as follows

$$\pi_1 = \omega_k \tau = \Phi_k, \quad \pi_2 K = \rho_D, \quad \pi_3 K = \rho_I, \quad \pi_4 = \vartheta, \qquad \frac{\pi_5}{\pi_4^2} = \lambda = \frac{a_0}{a_1^2}, \quad \pi_6 = t/\tau \tag{13}$$

Another kind of second-order dimensionless time delay model than (11) is proposed in [15] assuming a possibility of non-minimum-phase character due to transfer function zero but excluding the oscillatory type of behaviour.

The special case $a_0 = 0$, i.e. the integrating plant necessitates an essential modification of swingability number λ. Unlike (1) the plant model is considered in the form $y''(t) + a_1 y'(t) = b_0 u(t)$ and instead of λ the following similarity number

$$\lambda_I = \frac{b_0}{{a_1}^2}, \quad a_1 > 0, \quad b_0 > 0 \tag{14}$$

is introduced. In contrast to λ parameter λ_I has no connection with oscillations, it represents a ratio of both exponential and integration time constants. The corresponding dimensionless modification of (11) is as follows

$$\frac{d^2y(\bar{t})}{d\bar{t}^2} + \vartheta \frac{dy(\bar{t})}{d\bar{t}} = \vartheta^2 \lambda_I u(\bar{t} - 1), \qquad \vartheta > 0, \quad \lambda_I > 0 \tag{15}$$

3. Ultimate angle to swingability and laggardness relationship

Although the range of plants that can be satisfactorily PID controlled is limited the surprising versatility of this controller principle has undoubtedly been proved in its implementation. The close relationship between the *ultimate gain* and *frequency* and the PID controller setting is well known and particularly the assessment of them by applying the ideal relay feedback has led to the contemporary wide application of this approach [3]. With regard to dominant pole placement, it is important to notice that the ultimate frequency ω_k determines the bounds within which the attainable natural frequency of the control loop can be considered.

To determine the ultimate gain and the corresponding ultimate frequency for the plant (11) we need to get its characteristic equation. With regard to the introduced time ratio $\bar{t} = t/\tau$ in (11) the adequate modification of the Laplace transform operator $s \to \bar{s} = s\tau$ is adopted. Assuming there exists a feedback proportional gain K_k added in (11) the following characteristic equation is considered

$$\bar{s}^2 + \vartheta\bar{s} + \vartheta^2\lambda + K\vartheta^2\lambda K_k \exp(-\bar{s}) = 0 \tag{16}$$

As it resulted from the set of dimensionless parameters (13) the *ultimate angle*, $\Phi_k = \omega_k\tau$, is the selected similarity number for the ultimate frequency. According to the Buckingham's theorem a dimensionless relationship has to exist between Φ_k and the parameters ϑ, λ, K.

Theorem 1. *For any pair of the similar plants (11) with the same $\vartheta > 0$, $\lambda > 0$, K it holds that their ultimate angle Φ_k is also the same and satisfies the equation*

$$\tan \quad \Phi_k = \frac{\vartheta\Phi_k}{\Phi_k^2 - \vartheta^2\lambda} \to \Phi_k = \Phi_k(\lambda, \vartheta) \tag{17}$$

Proof. If the ultimate gain K_k in (16) is adjusted then the stability margin is reached and undamped oscillations at frequency ω_k arise. The dimensionless Laplace operator is $\bar{s} = j\omega_k\tau = j\Phi_k$ and therefore the following condition is obtained from (16)

$$-\Phi_k{}^2 + \vartheta\Phi_k + \vartheta^2\lambda + K\vartheta^2\lambda K_k \exp(-j\Phi_k) = 0 \tag{18}$$

which after decomposition results in two following equalities of the real and imaginary parts, i.e. $K\vartheta^2\lambda K_k \cos \Phi_k = \Phi_k{}^2 - \vartheta^2\lambda$, and $K\vartheta^2\lambda K_k \sin \Phi_k = \vartheta\Phi_k$. The ultimate gain K_k, K and $\vartheta^2\lambda$ on the left-hand sides can be excluded by means of evaluating the tangent function of Φ_k and Eq. (17) is obtained where the ultimate angle as $\Phi_k = \Phi_k(\lambda, \vartheta)$ can be determined from. The ultimate angle Φ_k is dependent only on λ, ϑ, the influence of K is cancelled. Owing to the periodicity of the tangent function equation (17) admits infinitely many real solutions. However, with respect to both the periodical character of tangent function and the physical meaning of the ultimate frequency only the smallest of the positive roots Φ_k of (17) can represent the ultimate angle and consequently the ultimate frequency ω_k as well. ■

When investigating the function $\Phi_k = \Phi_k(\lambda, \vartheta)$ it should be noted that in fact the value of the *laggardness number* ϑ can fall only into a rather narrow interval, approximately $\vartheta \in \langle 0.5, 3 \rangle$. The upper bound of this range is to exclude *too large delays.* The values $\vartheta < 3$ would bring about a situation when the classical PID controller feedback action unavoidably comes *too late* to compensate the impact of disturbances and obviously it is meaningless to deal with such options. More sophisticated control strategies than PID are to be applied in such cases. The lower bound of ϑ prevents model (8) from its application to a plant with too small delay. Too small value of $\vartheta(\vartheta < 0.5)$ indicates that the delay is a marginal property of the plant and then it is suitable model (8) to replace by a delay-free model.

Also the values of the *swingability number* λ are to be limited. The upper bound $\lambda \leq 2$ is considered to exclude the plants with excessively weakly damped oscillations which are not the object of investigation here. On the other hand, the case of the *integrating plant,* $a_0 = 0$, needs to apply the special λ_I according to (14). Due to the missing term $a_0 y$ the relationship for the ultimate angle Φ_k simplifies itself to the form: $\tan \Phi_k = \vartheta/\Phi_k$ independent of λ_I. By the way, note that this form is also equal to the limit of (17) for $\lambda \to 0$. Owing to the constrained values of λ and ϑ the values of ultimate angle $\Phi_k = \Phi(\lambda, \vartheta)$ are also from a relatively narrow interval. If $\vartheta \in \langle 0.5, 3 \rangle$ and $\lambda \in \langle 0, 2 \rangle$ are considered then it holds that $\Phi_k \in \langle 0.6, 2.6 \rangle$ approximately. A three-dimensional plot of $\Phi_k = \Phi(\lambda, \vartheta)$ is in Fig. 1.

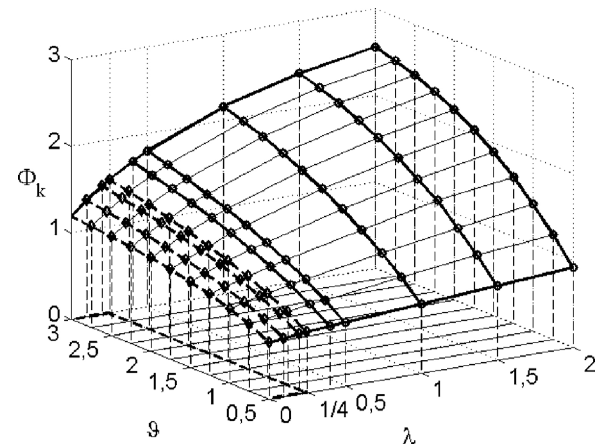

Fig. 1. Dimensionless relationship of ultimate angle Φ_k on λ and ϑ.

4. Three-pole placement by setting PID control loop parameters

Only three of the seven dimensionless variables introduced for a delayed PID control loop are independent, namely ϑ, λ as dynamical specification of the plant and $\bar{t}$. The remaining four are dependent on them as it results from the Buckingham's theorem and the first of these relationships we found as (17) for Φ_k. The remaining three dimensionless relationships for ρ_0, ρ_D, ρ_I are covered in this section. This intent is consistent with the aim to place just three poles $\bar{p}_{1,2,3}$ of the loop and as in the previous considerations it is necessary to distinguish the case of integrating plant, $a_0 = 0$ from the standard options of the plant, $a_0 > 0$.

An intuitive dimensionless approach to dominant three pole placement in PID control loop was already proposed by Hwang [16], although the delay effect was not explicitly considered and ω/ω_k was used as the primary independent variable. The idea of dominant pole placement itself was introduced by Hwang and Chang [1] where it is documented that the closed loop response is primarily dominated in most of PID control systems by a *trio of poles*: a complex conjugate pair and a real pole.

For the next consideration suppose a time delay plant in the dimensionless form (11) with $\lambda > 0$ and a PID controller as in (2). After applying the dimensionless parameters from (4) and (13) this controller is described by the dimensionless equation

$$K\frac{du}{d\bar{t}} = \rho_0\frac{de}{d\bar{t}} + \rho_D\frac{d^2e}{d\bar{t}^2} + \rho_I e \tag{19}$$

where ρ_0, ρ_D, ρ_I are the dimensionless gains of proportional, derivative and integration actions respectively, $\rho_0 = Kr_0$, $\rho_D = \frac{Kr_D}{\tau}$, $\rho_I = K\tau r_I$, and e is the control error.

Basically the pole placement is a matter of forming the characteristic quasi-polynomial of the control loop. After joining up the plant (11) with the controller (19) the third order differential equation of the control loop is obtained and in cases with $a_0 > 0$ its dimensionless characteristic quasi-polynomial is of the following form

$$\bar{M}(\bar{s}) = \bar{s}^3 + \vartheta\bar{s}^2 + \vartheta^2\lambda\bar{s} + \exp(-\bar{s})\vartheta^2\lambda[\rho_0\bar{s} + \rho_D\bar{s}^2 + \rho_I] \tag{20}$$

where none of the coefficients is zero. In Section 2 we found that the similarity of PID control loops composed of (1) and (2) is associated with the dimensionless parameters ϑ, λ, ρ_0, ρ_D, ρ_I and now we see that in the case of the same values of these parameters quasi-polynomial (20) is also of identical form for different but similar control loops. Quasi-polynomial $\bar{M}(\bar{s})$ then gives a uniform description of *all* the *dimensionally similar* control loops with the same ϑ, λ, ρ_0, ρ_D, ρ_I when $a_0 > 0$.

Remark. Assume a group of dimensionally similar controlled plants ($a_0 > 0$) as in (11) with identical values of ϑ, λ, K. If their particular PID controllers are set so that their dimensionless control loop parameters ρ_0, ρ_D, ρ_I are of the same values for each considered case then the responses of the control loops are identical when plotted in the time scale of their common dimensionless time $\bar{t}$ since the dimensionless model of all these control loops is *identical*, i.e. common for all of them.

The similar plants have not only the same *laggardness* and *swingability* numbers, due to (17) they also have the same ultimate angle Φ_k. In the next we will show the PID loop parameters ρ_0, ρ_D, ρ_I as dependent on ϑ, λ, Φ_k but just due to (17) their dependence only on ϑ and λ is to be considered. The main benefit of using the dimensionless parameters is that each of the dynamically similar control loops will result in *the same* quasi-polynomial (20). Therefore not only all the dimensionless parameters ϑ, λ, ρ_0, ρ_D, ρ_I but also all of the infinitely many dimensionless poles of $\bar{M}(\bar{s})$ spectrum are identical for any of dimensionally similar control loops.

The aim of the next considerations is to prescribe certain three numbers $\bar{p}_{1,2,3}$ as desirable dominant zeros of $\bar{M}(\bar{s})$. The close coherence between the *ultimate frequency* and the most natural frequency of control response is well known and the option $\Omega \cong \omega_k$ is commonly used as a rule of thumb by Hwang and Fang [4], or Åström and Hägglund [3]. The relationship between the prescribed frequency and ω_k was also applied by Wang et al. [17]. Any attempts to make Ω substantially higher than ω_k unavoidably lead to the loss of dominance for the prescribed pair and Ω substantially lower than ω_k results in a groundlessly sluggish response. Hwang and Fang [4] made a thorough search for IAE optimum placement of three poles with the imaginary part $\Omega \in \langle 0.9 - 1.1 \rangle \, \omega_k$ of the complex conjugate pair. With regard to these results we prescribe the natural frequency Ω of the control loop just equal to the ultimate frequency ω_k, i.e. the ultimate angle $\Phi_k = \omega_k \tau$ as the imaginary part of $\bar{p}_{1,2}$ in the case of dimensionless description of the control loop. Therefore we choose the following three numbers

$$\bar{p}_{1,2} = (-\delta \pm j)\Phi_k, \quad \bar{p}_3 = -\kappa\delta\Phi_k \tag{21}$$

as the prescribed dominant zeros of $\bar{M}(\bar{s})$, where δ is the *damping ratio* of $\bar{p}_{1,2}$ and κ is the *root ratio* $\kappa = |\bar{p}_3|/|Re\bar{p}_{1,2}|$. The following theorem holds for the relationship between these *three poles* of the control loop and its three control parameters ρ_0, ρ_D, ρ_I.

Theorem 2. *Consider a class of similar control loops with a common characteristic quasi-polynomial* (20) *with* $\lambda > 0$ *and three numbers* $\bar{p}_{1,2,3}$ *as in* (21) *selected to become the zeros of* $\bar{M}(\bar{s})$. *Let the ratios* δ, κ *be considered as parameters to be selected later. If* $\bar{p}_{1,2,3}$ *are assigned as* $\bar{M}(\bar{s})$ *zeros (so far without guaranteeing their dominance) the control loop parameters* ρ_0, ρ_D, ρ_I *are to be adjusted on the values given by the following explicit formulae*

$$\rho_0 = \frac{1}{1+\delta^2(\kappa-1)^2}\begin{vmatrix} B_1, & -(1-\delta^2), & 1 \\ B_2, & -2\delta, & 0 \\ B_3, & \kappa^2\delta^2, & 1 \end{vmatrix} \tag{22}$$

$$\rho_D = \frac{1}{\Phi_k\left[1+\delta^2(\kappa-1)^2\right]}\begin{vmatrix} -\delta, & B_1, & 1 \\ 1, & B_2, & 0 \\ -\kappa\delta, & B_3, & 1 \end{vmatrix} \tag{23}$$

$$\rho_I = \frac{\Phi_k}{1+\delta^2(\kappa-1)^2}\begin{vmatrix} -\delta, & -(1-\delta^2), & B_1 \\ 1, & -2\delta, & B_2 \\ -\kappa\delta, & \kappa^2\delta^2, & B_3 \end{vmatrix} \tag{24}$$

where

$$B_1 = \exp(-\delta\Phi_k)[b_R\cos\Phi_k - b_I\sin\Phi_k], \tag{25}$$

$$B_2 = \exp(-\delta\Phi_k)[b_R\sin\Phi_k + b_I\cos\Phi_k], \tag{26}$$

$$B_3 = \exp(-\kappa\delta\Phi_k)\left[\kappa\delta - \frac{1}{\lambda\vartheta}\kappa^2\delta^2\Phi_k + \frac{1}{\lambda\vartheta^2}\kappa^3\delta^3{\Phi_k}^2\right], \tag{27}$$

$$b_R = \delta + \frac{1}{\lambda\vartheta}(1-\delta^2)\Phi_k - \frac{1}{\lambda\vartheta^2}(3\delta-\delta^3){\Phi_k}^2, \tag{28}$$

$$b_I = -1 + \frac{1}{\lambda\vartheta}2\delta\Phi_k + \frac{1}{\lambda\vartheta^2}(1-3\delta^2){\Phi_k}^2 \tag{29}$$

Proof. After inserting $\bar{p}_1 = (-\delta + j)\Phi_k$ into equality $\bar{M}(\bar{s}) = 0$ and dividing by $\exp(-\bar{s})\vartheta^2\lambda$ we obtain

$$\begin{aligned}&\rho_0(-\delta+j)\Phi_k + \rho_D(\delta^2-1-j2\delta){\Phi_k}^2 + \rho_I \\ &= \exp(-\delta\Phi_k)(\cos\Phi_k + j\sin\Phi_k)\left[(\delta-j)\Phi_k + \frac{1}{\lambda\vartheta}(1-\delta^2+j2\delta){\Phi_k}^2 \right. \\ &\left. -\frac{1}{\lambda\vartheta^2}(3\delta-\delta^3-j(1-3\delta^2)){\Phi_k}^3\right] = \tilde{B}\end{aligned} \tag{30}$$

Using the expressions b_R and b_I given in (28) and (29), the real and imaginary parts of $\tilde{B}$, respectively, may be expressed as follows

$$\begin{aligned}Re(\tilde{B}) &= B_1\Phi_k = \exp(-\delta\Phi_k)[b_R\cos\Phi_k - b_I\sin\Phi_k]\Phi_k \\ Im(\tilde{B}) &= B_2\Phi_k = \exp(-\delta\Phi_k)[b_R\sin\Phi_k + b_I\cos\Phi_k]\Phi_k\end{aligned} \tag{31}$$

Inserting the third pole $\bar{p}_3 = -\kappa\delta\Phi_k$ into $\bar{M}(\bar{s}) = 0$ we obtain

$$\begin{aligned}&-\rho_0\kappa\delta\Phi_k + \rho_D(\kappa\delta\Phi_k)^2 + \rho_I = \exp(-\kappa\delta\Phi_k)\left[\kappa\delta\Phi_k - \frac{1}{\lambda\vartheta}(\kappa\delta\Phi_k)^2 \right. \\ &\left. + \frac{1}{\lambda\vartheta^2}(\kappa\delta\Phi_k)^3\right]\end{aligned} \tag{32}$$

From (30) and (32) the set of equations $\mathbf{A}{\cdot}\mathbf{P} = \mathbf{B}$ results with the following matrices

$$\mathbf{A} = \begin{bmatrix} -\delta\Phi_k, & -(1-\delta^2){\Phi_k}^2, & 1 \\ \Phi_k, & -2\delta{\Phi_k}^2, & 0 \\ -\kappa\delta\Phi_k, & (\kappa\delta)^2{\Phi_k}^2, & 1 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} \rho_0 \\ \rho_D \\ \rho_I \end{bmatrix} \tag{33}$$

$$\mathbf{B} = \begin{bmatrix} \Phi_k\exp(-\delta\Phi_k)(b_R\cos\Phi_k - b_I\sin\Phi_k) \\ \Phi_k\exp(-\delta\Phi_k)(b_R\sin\Phi_k + b_I\cos\Phi_k) \\ \Phi_k\exp(-\kappa\delta\Phi_k)\left[\kappa\delta - \frac{1}{\lambda\vartheta}(\kappa\delta)^2\Phi_k + \frac{1}{\lambda\vartheta^2}(\kappa\delta)^3{\Phi_k}^2\right] \end{bmatrix} = \begin{bmatrix} \Phi_k B_1 \\ \Phi_k B_2 \\ \Phi_k B_3 \end{bmatrix} \tag{34}$$

If the solution of the set $\mathbf{A}\ \mathbf{P} = \mathbf{B}$, is performed and reduced by the powers ${\Phi_k}^3$, ${\Phi_k}^2$, ${\Phi_k}^3$ in fractions for ρ_0, ρ_D, ρ_I respectively, formulae (22)–(24) are directly obtained.■

The results (22)–(24) lead to an important conclusion. If for two plants with *the same* λ, ϑ *the same zeros* $\bar{p}_{1,2,3}$ of the generalized $\bar{M}(\bar{s})$ are prescribed then also *the same* generalized PID *control gains* ρ_0, ρ_D, ρ_I lead to achieving them. However, as we expected above only a selection of certain specific suitable δ, κ can match the pole placement aim, i.e. $\bar{p}_{1,2,3}$ actually become the dominant

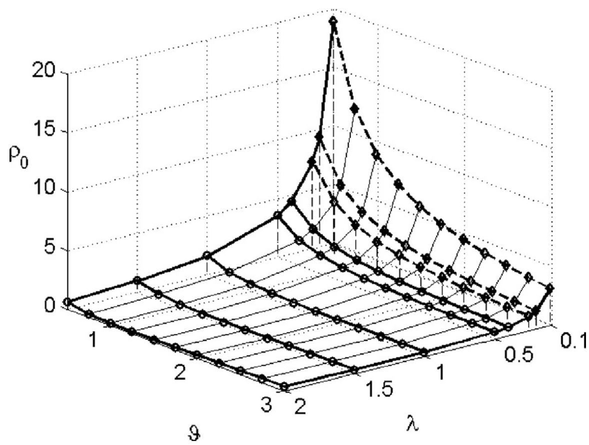

Fig. 2. Control loop *proportional* gain setting for $k = 1.3$ and $\delta = 0.35$ ($a_0 > 0$).

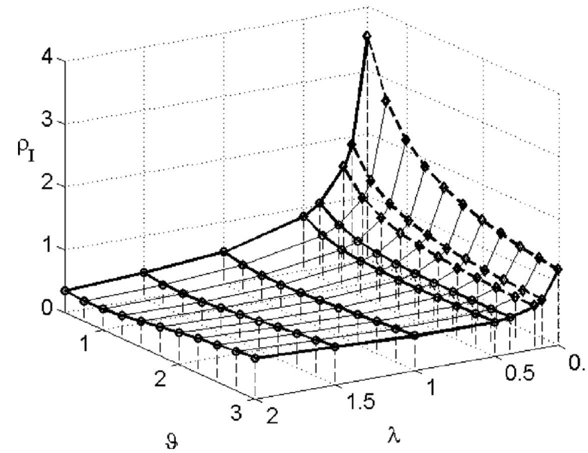

Fig. 4. Control loop *integration* gain setting for $k = 1.3$ and $\delta = 0.35$ ($a_0 > 0$).

poles of the particular control loop. Therefore a means for testing the dominance of the placed $\bar{p}_{1,2,3}$ is necessary in any application of (22)–(24). This task is solved in Sections 5 and 6.

The control gain setting given by (22)–(24) holds for all the considered plants when $a_0 > 0$. As we saw in the previous sections the case of the *integrating plant* with $a_0 = 0$ is to be treated with the modified dimensionless parameter $\lambda_I = b_0/a_1^2$ instead of λ. In the quasi-polynomial $\bar{M}(\bar{s})$ the $\bar{s}$ term is missing and instead of (20) the following form is to be considered

$$\bar{M}(\bar{s}) = \bar{s}^3 + \vartheta\bar{s}^2 + \exp(-\bar{s})\vartheta^2\lambda_I[\rho_0\bar{s} + \rho_D\bar{s}^2 + \rho_I] \tag{35}$$

It is easy to see that the modification of Theorem 2 for the case $a_0 = 0$ is only slight. First Eqs. (22) through (26) do not change at all and also matrix **A** remains the same. In Eqs. (27)–(29) the terms $\kappa\delta$, δ and -1, respectively, are omitted, and λ_I is substituted instead of each λ. However, the values of ρ_0, ρ_D, ρ_I obtained from this modification are different from those we obtain from (22)–(24) for the lowest values of λ, i.e. any consideration of $\lambda \to 0$ in the mentioned formulae, as in (17), leads to division by zero, i.e. is meaningless.

The relationships in Theorem 2 contain five input variables, where due to (17), $\Phi_k = \Phi(\vartheta, \lambda)$, only four of them, namely ϑ, λ, κ and δ, are mutually independent. If then the root and damping ratios κ, δ are considered as fixed constants, the control loop parameters ρ_0, ρ_D, ρ_I can be viewed as dependent on the plant parameters ϑ, λ only and can be displayed as 3D graphs. For the option of fixed $\kappa = 1.3$ and $\delta = 0.35$ and λ, ϑ ranging from 0.1 to 2 and from 0.5 to 3, respectively the settings ρ_0, ρ_D, ρ_I obtained from (22)–(24) are displayed in Figs. 2–4 for the plants with $a_0 > 0$. For the same option of $\kappa = 1.3$, $\delta = 0.35$ and for the same range of ϑ and λ_I, the settings for integrating plants with $a_0 = 0$ are in Figs. 5–7.

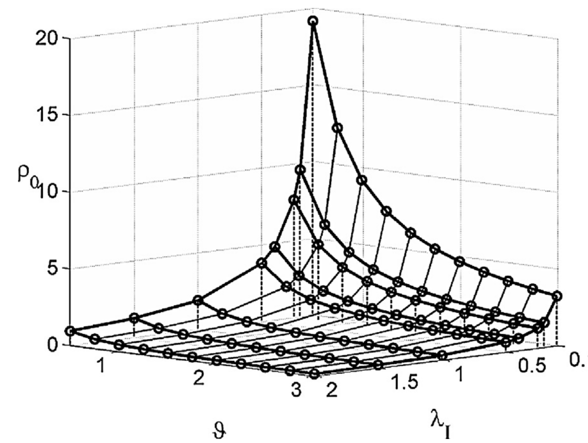

Fig. 5. *Proportional* gain setting with integrating plant for $k = 1.3$ and $\delta = 0.35$.

In Figs. 2 through 7 there is apparent the benefit brought about by the use o the similarity numbers. Into these graphs we have summarized *all* the pole-placement-based settings of PID control loops whose plants are identifiable with model (11) or (15). On the other hand, let be emphasized that formulae (22)–(24) have been obtained without any regard to the actual evidence of $\bar{p}_1$, $\bar{p}_2$, $\bar{p}_3$

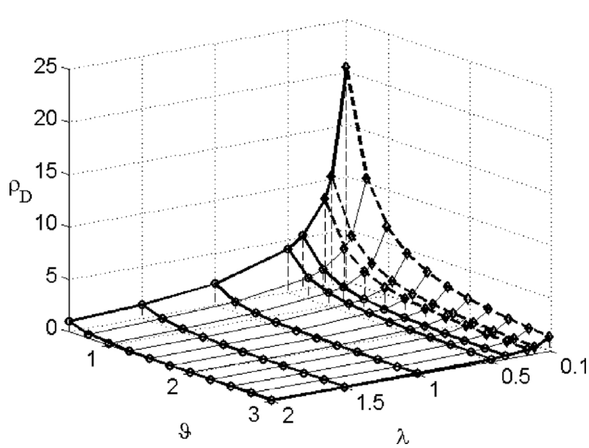

Fig. 3. Control loop *derivative* gain setting for $k = 1.3$ and $\delta = 0.35$ ($a_0 > 0$).

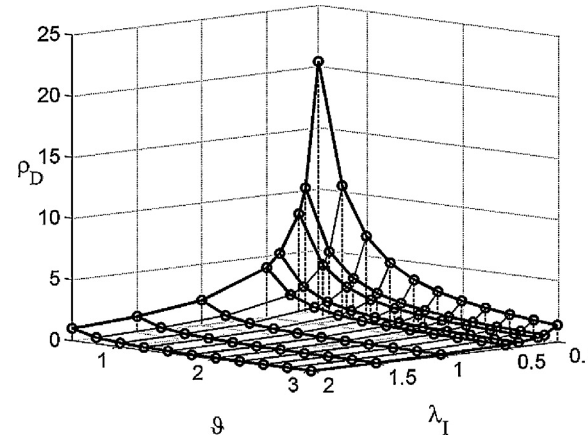

Fig. 6. *Derivative* gain setting with integrating plant for $k = 1.3$ and $\delta = 0.35$.

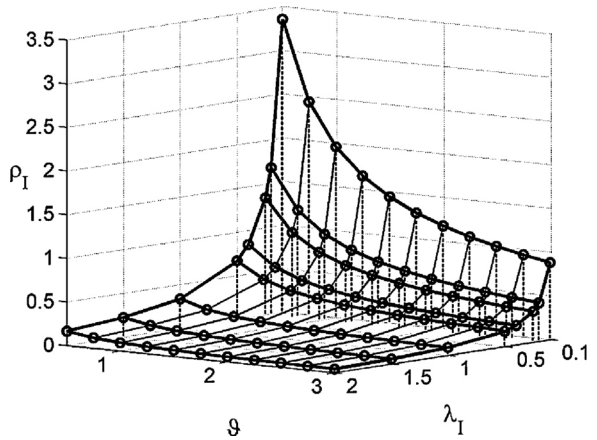

Fig. 7. *Integration* gain setting with integrating plant for k = 1.3 and δ = 0.35.

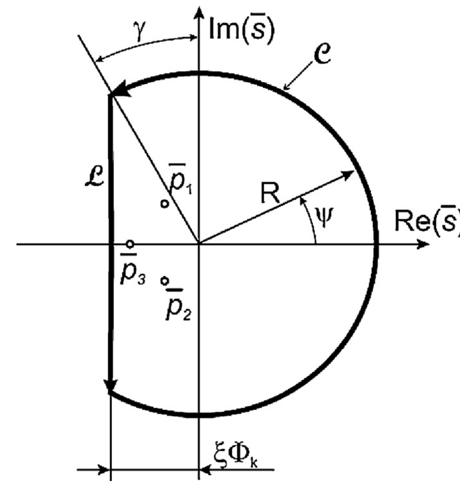

Fig. 8. The Jordan curve for the argument increment test.

dominance. If some improper values of κ, δ are prescribed the application of these formulae would lead to unacceptable settings owing to the loss of dominance. That is why the dominance must always be tested and its proof is the main issue of the rest of the paper. However, in spite of this problem the reader may rest assured that all the settings displayed in Figs. 2 through 7 correspond solely to *dominant pole placements*. During the simulations each of them was tested by the proofs presented in the next two sections.

5. Argument increment based test of dominance

Any of the poles $\bar{p}_{1,2,3}$ cannot be validated for the controller tuning (22)–(24) before a verification that they really assume the dominant position within the spectrum of the considered control loop. The dominance of $\bar{p}_{1,2,3}$ is inevitably connected with their unambiguously *rightmost position* within the whole control loop spectrum. On the one hand it is possible to compute a sufficient set of the rightmost poles by means of a root-finder tool [18]. But on the other hand the prescribed roots $\bar{p}_{1,2,3}$ are already given – only their sufficiently separate position to the right from the rest of the spectrum is to be checked for a large number of the control loop options. That is why the following dominance check of $\bar{p}_{1,2,3}$ is proposed.

Lemma 2. *Consider the characteristic quasi-polynomial (20) of the PID control loop and let $\bar{s}$ be fixed to the straight line L, $\bar{s} = -\xi\Phi_k + j\varphi$, parallel to the Im axis, where $\xi > \kappa\delta$. If the argument increment of $\bar{M}(\bar{s})$ along L from the starting point $\bar{M}(-\xi\Phi_k)$, $(\varphi = 0)$, for φ growing from zero to infinity, $\varphi \to \infty$, reaches the following limit*

$$\lim_{\varphi\to\infty} \Delta arg\bar{M}(\bar{s})|_{\bar{s}=-\xi\Phi_k+j\varphi} = -3\frac{\pi}{2} \tag{36}$$

then the whole rest of $\bar{M}(\bar{s})$ spectrum except $\bar{p}_{1,2,3}$ lies to the left of the straight line L.

Proof. Assume that only the poles $\bar{p}_{1,2,3}$ lie *inside* a region as in Fig. 8 enclosed by a Jordan curve composed of

- a circle-arc C of radius R, $\bar{s} = R\exp(j\psi)$, where ψ ranges from $-\pi/2 - \gamma$ to $\pi/2 + \gamma$, where $\gamma = \arcsin(\xi\Phi_k/R)$,
- and a straight line L, $\bar{s} = -\xi\Phi_k + j\varphi$, $\quad \xi > \kappa\delta$ parallel to the imaginary axis where φ ranges from $-R\cos\gamma$ to $R\cos\gamma$.

If just only $\bar{p}_{1,2,3}$ of $\bar{M}(\bar{s})$ zeros are enclosed by C and L the total *argument increment* along C and L has to be 6π. To evaluate this argument increment by parts for C and L let $\bar{M}(\bar{s})$ be factorized as follows

$$\bar{M}(\bar{s}) = \bar{s}^3 m(\bar{s}) = \bar{s}^3\left[1 + \frac{\vartheta}{\bar{s}} + \frac{\vartheta^2\lambda}{\bar{s}^2} + \exp(-\bar{s})\vartheta^2\lambda\left(\frac{\rho_0}{\bar{s}^2} + \frac{\rho_D}{\bar{s}} + \frac{\rho_I}{\bar{s}^3}\right)\right] \tag{37}$$

For the first factor $\bar{s}^3$ the argument increment along C is obvious

$$\underset{C}{\Delta} arg\bar{M}(R\exp(j\psi)) = 6\left(\frac{\pi}{2} + \gamma\right) \tag{38}$$

where γ approaches zero for $R \to \infty$, and therefore the limit of this increment for $R \to \infty$ is 3π. For the second factor $m(\bar{s})$ with $\bar{s} = R\exp(j\psi)$, the values of each of its terms except 1 are vanishing for $R \to \infty$ (due to the powers of R in the denominator) and therefore $\lim_{R\to\infty} m(R\exp(j\psi)) = 1$ and its argument increment approaches zero for $R \to \infty$. Hence the whole argument increment of $\bar{M}(R\exp(j\psi))$ along C is given by (38) and for $R \to \infty$ it is 3π. If there are just three $\bar{M}$ zeros $\bar{p}_{1,2,3}$ inside the considered region the argument increment along L has to be given by the difference $6\pi - 3\pi = 3\pi$, if L is oriented downwards as in Fig. 8. Finally, with respect to the symmetry, if for practical purposes the original interval $\varphi \in \langle R\cos\gamma, -R\cos\gamma\rangle$ is replaced by only its positive half, oriented *upwards*, $\varphi \in \langle 0, R\cos\gamma\rangle$ the required argument increment is of half value and of opposite sign, i.e.

$$\lim_{\varphi\to\infty} \Delta arg\bar{M}(-\xi\Phi_k + j\varphi) = -3\frac{\pi}{2} \tag{39}$$

as in (36).■

The application of this criterion is as follows. For κ and δ in $\bar{p}_{1,2,3}$ a value of $\xi > \delta\kappa$ is chosen, and $\bar{M}(-\xi\Phi_k + j\varphi)$ is computed for φ growing from zero to some $\varphi_m \gg \Phi_k$. In principle, the starting real value of $M(-\xi\Phi_k)$ for $\varphi = 0$ may happen to be both negative and positive for various types and orders of $\bar{M}(\bar{s})$. Nevertheless for each of the third-order $\bar{M}(\bar{s})$ investigated here this value is negative $M(-\xi\Phi_k) < 0$. The argument increment is evaluated in an analogous way as in the Mikhaylov criterion application. If the $\bar{M}(-\xi\Phi_k + j\varphi)$ contour winds up by $-3\pi/2$ in the clockwise direction for $\varphi \to \infty$ the dominance of $\bar{p}_{1,2,3}$ is verified while otherwise it is lost. The ratio $\xi > \delta\kappa$ is optional, the higher ξ for which the condition (36) is still satisfied, the stronger the dominance of $\bar{p}_{1,2,3}$. The highest value of $\xi = \xi_m$ for which the condition (36) is still fulfilled is a measure of the distance between the dominant $\bar{p}_{1,2,3}$ and the rest of the spectrum, i.e. the degree of dominance. Therefore ratio $\xi_m/(\kappa\delta)$ is further referred to as *dominance index*.

In order to prevent the assessment of ρ_0, ρ_D, ρ_I from a loss of dominance it is advisable to follow each application of (22)–(24) by a test on the argument increment. An example of evaluating the argument increment of $\bar{M}(-\xi\Phi_k+j\varphi)$ is in Fig. 13a. The dominance index values of the placed poles for all the considered plants are in Fig. 12.

6. Selecting the root and damping ratios to guarantee the dominance

In Section 4 the control parameters ρ_0, ρ_D, ρ_I were assessed as if the ratios κ, δ were kept on given constant values and only λ, ϑ were considered variables. In this section the objective is to select the most fitting values of κ, δ ratios from the point of view of $\bar{p}_{1,2,3}$ dominance. These ratios are to be selected carefully because their unsuitable values can completely spoil the pole assignment result due to the loss of $\bar{p}_{1,2,3}$ dominance. Typically, for instance, the growth of κ beyond a limit unavoidably leads to some undesirable poles arising between $\bar{p}_{1,2}$ and $\bar{p}_3$ or even in the rightmost position. Also exceedingly high values of the damping ratio, $\delta>0.5$, lead to the loss of $\bar{p}_{1,2,3}$ dominance. Such a kind of failure is detectable by the test on argument increment (36).

In the following we consider the *disturbance rejection* as the evaluated performance property of the investigated control loop. After adding the disturbance term $K_d a_0 d(t-\tau)$ into plant Eq. (1) the corresponding modification of the dimensionless plant Eq. (11) is of the form

$$\frac{d^2y(\bar{t})}{d\bar{t}^2}+\vartheta\frac{dy(\bar{t})}{d\bar{t}}+\lambda\vartheta^2y(\bar{t})=K\lambda\vartheta^2u(\bar{t}-1)+K_d\lambda\vartheta^2d(\bar{t}-1),$$
$$\lambda>0,\quad \vartheta>0,\quad K,\quad K_d\neq 0 \tag{40}$$

where K_d, d are dimensionless again and the time delay is supposed the same for both u and d. Then after introducing the dimensionless parameters as in Section 2, for the case $a_0>0$, the transfer function of the loop for d is as follows

$$\bar{G}_{dM}(\bar{s})=\frac{K_d\bar{s}\,\exp(-\bar{s})}{\frac{1}{\lambda\vartheta^2}\bar{s}^3+\frac{1}{\lambda\vartheta}\bar{s}^2+\bar{s}+\exp(-\bar{s})[\rho_0\bar{s}+\rho_D\bar{s}^2+\rho_I]} \tag{41}$$

so that its spectrum of poles is *infinite*. Let the performance of the disturbance rejection be evaluated by the *absolute error integral* value (IAE)

$$I_{AE}=\int_0^\infty\left|e_d(\bar{t})\right|d\bar{t} \tag{42}$$

where $e_d(t)$ is the control error further considered as brought about by a step change of $d(t)$. The convergence of the improper integral (42) results from the integration action of the controller.

There are two ways the varying κ and δ influence the IAE value. Primarily these ratios affect just the modes appropriate to the placed poles $\bar{p}_1,\bar{p}_2,\bar{p}_3$, i.e. $\exp(-\kappa\delta\Phi_k\bar{t})$, $\exp(-\delta\Phi_k\bar{t})\cos\Phi_k\bar{t}$ and $\exp(-\delta\Phi_k\bar{t})\sin\Phi_k\bar{t}$, but besides the κ, δ values also influence the positions of *other poles* and their contribution to the control loop response and in this way they may possibly *deteriorate* the dominant position of $\bar{p}_1,\bar{p}_2,\bar{p}_3$. The following *auxiliary transfer function* is introduced for the sake of distinguishing these two impacts of κ, δ.

Proposition 1. *Consider the disturbance transfer function (41) as a result of assigning the poles $\bar{p}_1,\bar{p}_2,\bar{p}_3$ in PID control loop on the plant (11) with given λ, ϑ. Suppose that for this case various settings of κ, δ with corresponding ρ_0, ρ_D, ρ_I given by (22)–(24) are tried and tested by the criterion (42). For each of these options of (41) the poles $\bar{p}_1,\bar{p}_2,\bar{p}_3$ are assessed and the following auxiliary transfer function*

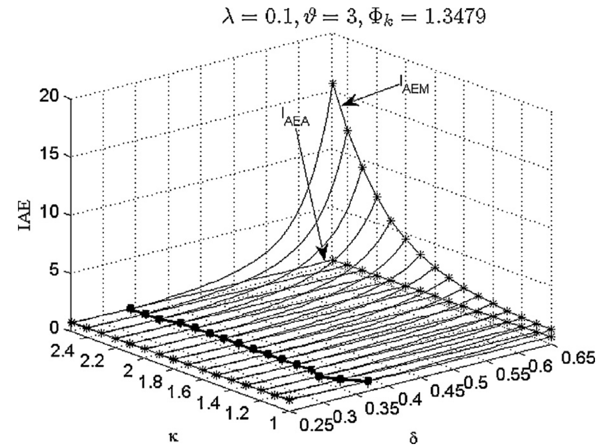

Fig. 9. I_{AEM} and I_{AEA} criteria of transfer functions (41) and (43) for a non-oscillating case of (11).

$$\bar{G}_{dA}(\bar{s})=\frac{K_d\bar{s}(-\bar{p}_1\bar{p}_2\bar{p}_3)\exp(-\bar{s})}{(\bar{s}-\bar{p}_1)(\bar{s}-\bar{p}_2)(\bar{s}-\bar{p}_3)\rho_I} \tag{43}$$

then represents a virtual control loop with only the three prescribed poles $\bar{p}_1,\bar{p}_2,\bar{p}_3$. Both the functions (41) and $\bar{G}_{dA}(\bar{s})$ have in common not only the prescribed poles and the dead time τ but, primarily, their step responses e_{dM} and e_{dA} for $\bar{G}_{dM}(\bar{s})$ and $\bar{G}_{dA}(\bar{s})$, respectively, have equal values of the following integrals $\int_0^\infty e_{dM}(\bar{t})d\bar{t}=\int_0^\infty e_{dA}(\bar{t})d\bar{t}=K_d/\rho_I$. As (43) represents a pattern of a function with only three poles $\bar{p}_1,\bar{p}_2,\bar{p}_3$ the difference between the I_{AE} integrals for both $e_{dM}(t)$ and $e_{dA}(t)$

$$\Delta I_{AE}=\int_0^\infty[\left|e_{dM}(t)\right|-\left|e_{dA}(t)\right|]dt==I_{AEM}-I_{AEA} \tag{44}$$

provides a quantity evaluating the influence of the rest of the spectrum beyond the prescribed $\bar{p}_1,\bar{p}_2,\bar{p}_3$ on the control loop response. The smaller the ΔI_{AE} value the stronger the dominance of $\bar{p}_1,\bar{p}_2,\bar{p}_3$, and vice versa.

Proof. Apparently the auxiliary $\bar{G}_{dA}(\bar{s})$ function has been constructed to fulfil the equality of its IAE integral with that of the "true" $\bar{G}_{dM}(\bar{s}):\int_0^\infty e_{dM}(\bar{t})d\bar{t}=\int_0^\infty e_{dA}(\bar{t})d\bar{t}=K_d/\rho_I$ (these integrals have no purpose in the response evaluation). Both the transfer functions result from identical prescription of $\bar{p}_1,\bar{p}_2,\bar{p}_3$ so that both their responses contain the modes having originated from these poles. By evaluating and comparing both the IAE integrals for various values of κ,δ in $p_{1,2,3}$ prescription one can follow the influence of κ,δ on the response in Figs. 9 and 10. While integral (42) is strongly dependent on κ and δ, the same integral for the auxiliary model (43) varies very weakly with κ,δ. Two examples of the dependence of both the integrals $I_{AEM}=\int_0^\infty\left|e_{dM}(\bar{t})\right|d\bar{t}$ and $I_{AEA}=\int_0^\infty\left|e_{dA}(\bar{t})\right|d\bar{t}$ on κ and δ are plotted in Figs. 9 and 10. For a relatively small set of κ and δ both these integrals are very close to each other so that for these options of $p_{1,2,3}$ their dominance is very strong and therefore the undesirable poles and modes have a negligible influence on the control loop response. On the contrary, for growing κ and δ the difference ΔI_{AE} increases considerably and its growing value cannot *originate in nothing else but in the undesirable influence* of the rest of the spectrum beyond the prescribed $\bar{p}_1,\bar{p}_2,\bar{p}_3$.■

In comparing the models (41) and (43) the difference ΔI_{AE} increases with the growing deficiency or even a loss of $\bar{p}_1,\bar{p}_2,\bar{p}_3$ dominance and its value can be used as an indicator of the *degree of the dominance*. The difference ΔI_{AE} is evaluated as a criterion for selecting the proper values of κ and δ for various options of λ and ϑ considered as *fixed* during each selection of optimum κ and δ. Unlike the *dominance index* $\xi_m/(\kappa\delta)$ from Section 5 the ΔI_{AE} assessment provides a possibility to select the proper values of the ratios

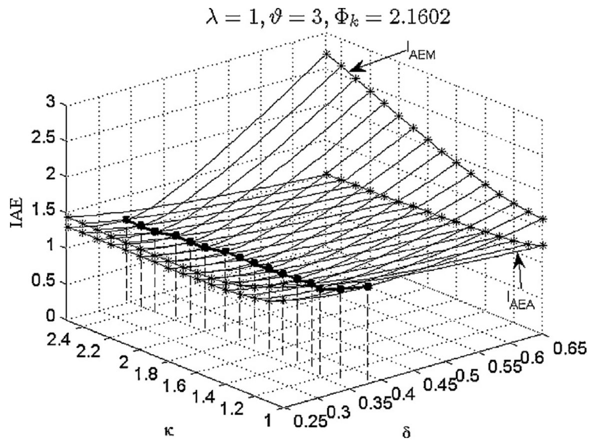

Fig. 10. I_{AEM} and I_{AEA} criteria of transfer functions (41) and (43) for an oscillating case of (11).

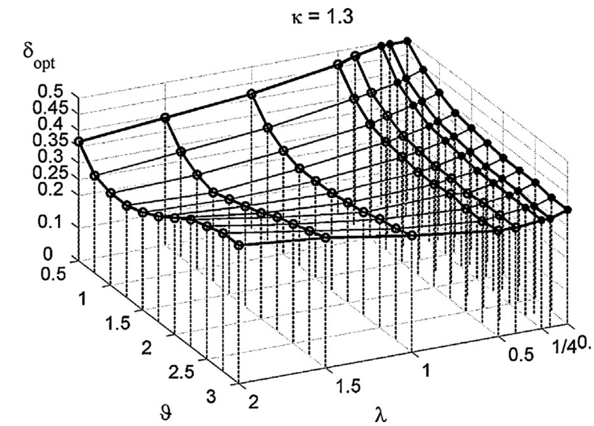

Fig. 11. Optimum values of damping ratio δ for the whole range of λ and ϑ.

κ and δ via finding the ΔI_{AE} *minimum*. Only those κ, δ which result in a minimum value of ΔI_{AE}, can be considered as the acceptable root and damping ratios for the dominant pole placement.

The graphs of I_{AEM} and I_{AEA} criteria for two samples of plants with $\lambda = 0.1$, $\vartheta = 3$ (aperiodic) and $\lambda = 1$, $\vartheta = 3$ (oscillating) are in Figs. 9 and 10, respectively. The dependence on κ, δ is mapped for the ranges $\delta \in \langle 0.25, 0.65\rangle$, $\kappa \in \langle 1.0, 2.5\rangle$. The upper graphs represent the true I_{AEM} and the lower ones the auxiliary integral I_{AEA}. At a glance we notice that for almost the same pairs of κ, δ both the surfaces I_{AEM} and I_{AEA} coincide with each other approximately and in this way indicate the *best dominance* of $\bar{p}_1, \bar{p}_2, \bar{p}_3$. The coincidence of the graphs also proves true the assumption that model (43) *really can fit* the meromorphic one (41). Unlike the true I_{AEM} the auxiliary integral I_{AEA} is very weakly dependent on both the ratios κ, δ, it varies only less than ten per cent throughout their whole ranges. In contrast, the true integral I_{AEM} corresponding to the meromorphic transfer function (41) increases steeply as soon as higher values of δ are taken and this increase is the higher the higher κ is chosen.

So far the criterion ΔI_{AE} was applied without a regard to the argument criterion (36). Particularly as to the *root ratio* κ this condition is to be regarded too. As we see in Figs. 9 and 10, the difference ΔI_{AE} in its minimum values practically does not change within the interval $\kappa \in \langle 1.3, 2\rangle$, but in spite of this the values $\kappa > 1.5$ are not admissible for the pole placement. It is because at least one pole arises between $\bar{p}_{1,2}$ and $\bar{p}_3$ in this case and the *dominance index* condition (36) is not satisfied any more. For this reason a compromise option of about $\kappa \cong 1.3$ is to be recommended for $\bar{p}_3$.

The mappings of I_{AEM} and I_{AEA} like those in Figs. 9 and 10 were performed for a set of about 50 various pairs of similarity numbers λ and ϑ. The most important finding from all these investigations is the following: Although the shapes of the IAE criterion graphs differ from each other to some extent, the position of the minimum difference ΔI_{AE} *varies very slightly for various* λ and ϑ. For the damping ratio δ this important finding is illustrated in Fig. 11 where the δ *optimum values* for the considered region of λ and ϑ are plotted. In spite of largely different plants represented by the considered area of λ and ϑ the optimum values of δ are within a narrow interval $\delta \in \langle 0.3, 0.45\rangle$. For any PID controller by means of (22)–(24) we can pick up the optimum δ for prescribing $\bar{p}_{1,2,3}$ as in (21) for the considered plant parameters λ and ϑ from Fig. 11. On the other hand, if we compare Fig. 11 with Figs. 9 and 10 we notice that near the optimum the difference ΔI_{AE} is very little sensitive to small changes of κ, δ so that the dominance turns out to be *robust*. If we recall Fig. 2 through 7 where the ratios $\delta = 0.35$ and $\kappa = 1.3$ were fixed we find out that the obtained settings for all the plants appropriate to the displayed λ and ϑ represent a quasi-optimum controller setting in fact.

Remark. Although the graphs in Figs. 9 and 10 are displayed for the whole range of λ and ϑ any use of model (43) instead of (41) is admissible only near the optimum κ, δ where the dominance is proved. For κ, δ values beyond the optimum spot function (43) becomes meaningless as a model of the control loop in fact. Note that the rightmost part of the $\bar{M}(\bar{s})$ spectrum may also be assessed using the algorithm presented in [18].

We already learned that the criterion of ΔI_{AE} is not to be used as a single point of view. Hence for a definite proof of the proposed κ, δ setting for the whole range of considered plants the *dominance index* $\xi_m/(\kappa\delta)$ has been evaluated. The plot of its values is in Fig. 12 from where it is apparent that the dominance index of $\bar{p}_{1,2,3}$ is higher than 1.2 all over the whole range of λ and ϑ when $\kappa = 1.3$ and δ optimum from Fig. 11 is used for controller setting according to (22)–(24).

7. Application example of the pole placement and controller setting

To demonstrate the presented approach to dominant pole placement consider a plant given by the original equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{3}\frac{dy(t)}{dt} + y(t) = 0.5u(t - \sqrt{3}) \tag{45}$$

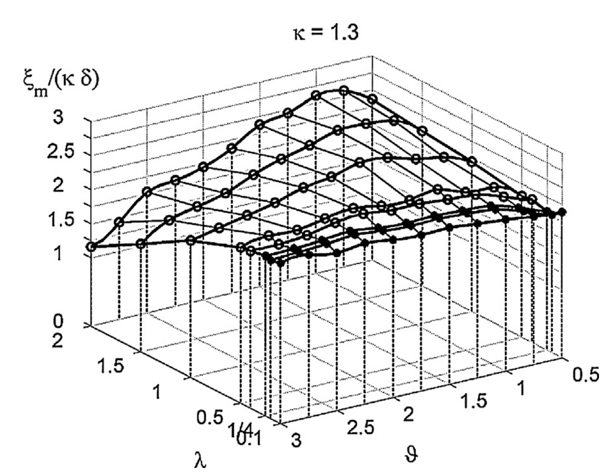

Fig. 12. Dominance index $\xi_m/(k\delta)$ values for various types of plants.

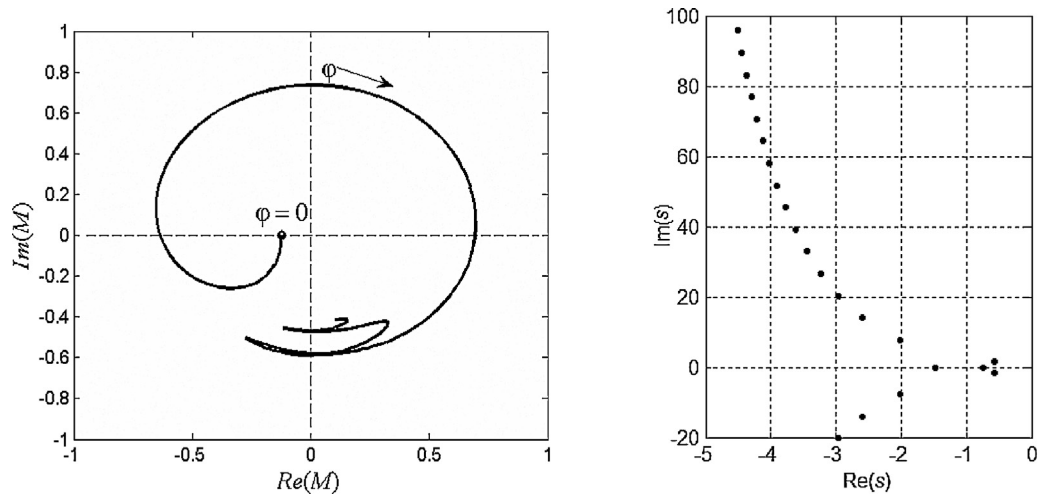

Fig. 13. (a) Argument increment checking the dominance of placed poles for (47), (b) the right-most part of spectrum of (47).

The similarity numbers of this plant are as follows $\lambda = 1/3$, $\vartheta = 3$, $K = 0.5$, and the dimensionless plant equation is as follows

$$\frac{d^2y(\bar{t})}{d\bar{t}^2} + 3\frac{dy(\bar{t})}{d\bar{t}} + 3y(\bar{t}) = 1.5u(\bar{t} - 1) \tag{46}$$

where $\bar{t} = t/\sqrt{3}$. The ultimate angle Φ_k is obtained as the root of (17) which for this case has the value $\Phi_k = 1.63635$. (The ultimate frequency of the plant is $\omega_k = 0.9447\ \mathrm{s}^{-1}$.) Using Φ_k as the imaginary part of the prescribed roots and the above recommended values of the root and damping ratios $\kappa = 1.3$, $\delta = 0.35$, respectively, the following dominant pole selection results: $\bar{p}_{1,2} = -0.5727 \pm j1.6364$ and $\bar{p}_3 = -0.7445$. Applying formulae (22)–(24) the following generalized control loop parameters are obtained $\rho_0 = 0.8726$, $\rho_D = 0.3489$ and $\rho_I = 0.6118$. The generalized quasi-polynomial (20) resulting from these parameters is as follows

$$\bar{M}(\bar{s}) = \bar{s}^3 + 3\bar{s}^2 + 3\bar{s} + 3\exp(-\bar{s})[0.8726\bar{s} + 0.3489\bar{s}^2 + 0.6118] \tag{47}$$

The spectrum of this quasi-polynomial is infinite and the prescribed $\bar{p}_{1,2,3}$ is the dominant group of this spectrum. The rightmost pole of the rest of the spectrum is real $\bar{p}_4 = -1.478$ with the position twice as far from the *Im* axis than $\bar{p}_3$ and then the spectrum continues by an infinite chain of $\bar{M}(\bar{s})$ zeros beginning with $\bar{p}_{5,6} = -2.01 \pm j1.64$, over, e.g. $\bar{p}_{35,36} = -4.52 \pm j95.78$, and so on, Fig. 13b [18]. The contribution of the whole rest of poles to the control loop behaviour is completely negligible. The condition (36) for the $\bar{M}(\bar{s})$ zeros $\bar{p}_{1,2,3}$ was also applied and the obtained contour $\bar{M}(-\xi\Phi_k + j\varphi)$ for the interval $\varphi \in \langle 0, 10\Phi_k \rangle$ with $\xi = 1.9\ \kappa\delta = 0.865$ is in Fig. 13a. The *dominance index* $\xi_m/(\kappa\delta) = 1.9$ confirms that the rightmost root of the rest of the spectrum has almost double distance from the imaginary axis than $\bar{p}_3$ so that the obtained dominance is very strong as can be seen from Fig. 13b. For drawing the contour in this graph instead of $\bar{M}(\bar{s})$ the following modification was applied

$$\bar{M}_P(\bar{s}) = \frac{\bar{M}(\bar{s})}{1 + \left|\bar{M}(\bar{s})\right|^{1.1}}, \quad \bar{s} = -\xi\Phi_k + j\varphi \tag{48}$$

which does not change the $arg\bar{M}(\bar{s})$ but reduces the module, $|\bar{M}_p(\bar{s})| < |\bar{M}(\bar{s})|$ [19]. The obtained control loop similarity numbers ρ_0, ρ_D, ρ_I correspond to the following final controller setting values

$$r_0 = \frac{\rho_0}{K} = 1.745, \quad r_D = \frac{\rho_D\tau}{K} = 1.209, \quad r_I = \frac{\rho_I}{K\tau} = 0.707 \tag{49}$$

So the prescribed $\bar{p}_{1,2,3}$ determine the response of the loop with a negligible influence of the rest of the spectrum, Fig. 14.

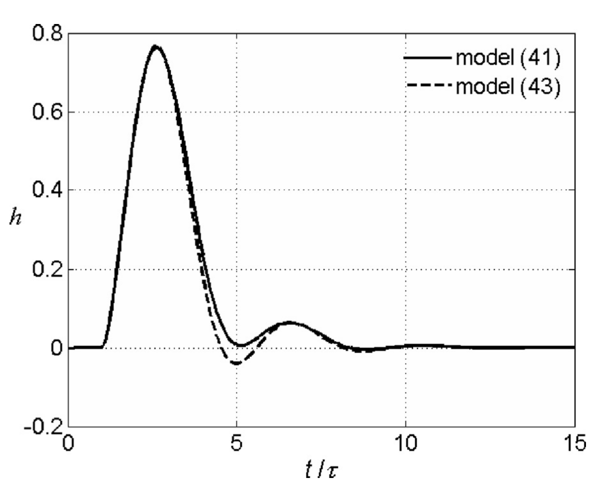

Fig. 14. The step responses of control loop for the setting (49) in model (41) or in the auxiliary (43).

8. Conclusions

The PID controllers still remain the most widely applied in the industry and the dominant pole placement in their tuning attract the research attention even though various specific controller schemes have been developed for the time delay systems, e.g. [20]. The original aim of this paper was to guarantee the dominance in the pole assignment. But the primary concern turns out to be in the matter of dimensionless treatment of the control loop dynamics where it is just as beneficial as it was proved in the traditional branches of science like hydrodynamics, thermodynamics, etc. The presented dimensionless investigation of the pole placement in the delayed PID control loop revealed the substance of the relationship between the specification of the prescribed poles $\bar{p}_{1,2,3}$ and the possibility really to achieve their dominant position. The presented formulae (22)–(24) with regard to the relationship (17) prove that the control parameters ρ_0, ρ_D, ρ_I can be considered as dependent only on four primary variables, namely two plant parameters *swingability* λ and *laggardness* ϑ and two ratios κ and δ of the prescribed pole specification. Hence the ultimate angle Φ_k is not included in this group because of its dependence on λ, ϑ.

However, from Section 6 we learned that the optimum dominance of $\bar{p}_{1,2,3}$ is obtained if the root and damping ratios κ and δ are set as follows, $\kappa \cong 1.3 \pm 0.1$ and $\delta \cong 0.35 \pm 0.05$, respectively, more details on δ setting are specified in Fig. 11. Just the results of Section 6 help arrive at the conclusion that in fact, the PID setting according to (22)–(24) can be considered as uniform, i.e. that ρ_0, ρ_D, ρ_I can be joined only with the given specification of the plant by λ and ϑ just as they are already displayed in Fig. 2 through 7. In fact the root and damping ratios κ and δ respectively may be kept constant, namely $\kappa = 1.3$ and $\delta = 0.35$. As it follows from Figs. 9 and 10 this fixation is acceptable because both the displayed criteria are fairly *insensitive* to small deviations of κ and δ in the vicinity of these values. In other words the choice of $\bar{p}_{1,2,3}$ connected with the ultimate angle Φ_k makes the control loop performance only very weakly dependent on κ and δ, as long as the dominance of $\bar{p}_{1,2,3}$ is kept sufficient.

Two novel tools for investigating the pole dominance are developed in the paper. First the argument increment criterion (36) or the dominance index are to prove the distinctly rightmost position of the placed poles $\bar{p}_{1,2,3}$. Another aspect of the dominance degree is introduced by applying the auxiliary finite spectrum model (43) to estimate the contribution of undesirable poles different from the prescribed $\bar{p}_{1,2,3}$ to the control loop response by means of the absolute difference ΔI_{AE}. Just this approach was decisive in clarifying the mechanism of the dominance decay due to an improper prescription of $\bar{p}_{1,2,3}$.

Acknowledgements

The presented research results were supported by The Technology Agency of the Czech Republic under the Competence Centre Project TE01020197, Centre for Applied Cybernetics 3.

Appendix. Identification of the dimensionless parameters

Model (11) has been chosen for describing the plant properties for the purpose of the PID controller adjustment. In this Appendix an additional relationship between the λ, ϑ parameters and the integrals of the step response $h(\bar{t})$ are presented to outline the ability of model (11) to be identified with a process given by its $h(\bar{t})$. The following Lemma holds for this identification.

Lemma 3. *Suppose that a stable SISO time delay process (1) with* $a_0 > 0$, $a_1 > 0$, $b_0 > 0$, $\tau > 0$ *is given by its unit-step response* $h(\bar{t})$, *i.e. in scaling where h is dimensionless and time is replaced by* $\bar{t} = t/\tau$. *For the dimensionless model (11) we have to find such parameters* K, λ, ϑ *that make* $h(\infty)$ *and the values of the following improper integrals*

$$h_I(\bar{t}) = \int_0^\infty [h(\infty) - h(\bar{t})]d\bar{t} \quad \text{and} \quad h_{II}(\bar{t}) = \int_0^\infty [h_I(\infty) - h_I(\bar{t})]d\bar{t} \tag{50}$$

the same for $h(\bar{t})$ and for the model. The model parameters satisfying this condition are as follows

$$K = h(\infty), \quad \vartheta = \frac{S_1}{{S_1}^2 + S_1 - S_2}, \quad \lambda = \frac{{S_1}^2}{{S_1}^2 + S_1 - S_2} \tag{51}$$

where $S_1 = h_I(\infty)/h(\infty) - 1$ and $S_2 = h_{II}(\infty)/h(\infty) - 0.5$.

Proof. Consider model (11) multiplied by $(\lambda\vartheta^2)^{-1}$, i.e.

$$\frac{1}{\lambda\vartheta^2}\frac{d^2y(\bar{t})}{d\bar{t}^2} + \frac{1}{\lambda\vartheta}\frac{dy(\bar{t})}{d\bar{t}} + y(\bar{t}) = Ku(\bar{t} - 1) \tag{52}$$

and suppose integrations (50) to be performed in the Laplace transform corresponding to $\bar{t}$, i.e. in the complex variable $\bar{s} = s\tau$. Let the dimensionless step response be $h(\bar{t})$ and its transform $h(\bar{t}) \to \bar{H}(\bar{s})$

$$\bar{H}(\bar{s}) = \frac{K\exp(-\bar{s})}{\bar{s}(\bar{s}^2(\lambda\vartheta^2)^{-1} + \bar{s}(\lambda\vartheta)^{-1} + 1)} \tag{53}$$

with the limit $\bar{t} \to \infty$ as $h(\infty) = \lim_{\bar{s}\to 0} \bar{s}\bar{H}(\bar{s}) = K$. The Laplace transforms of integrals (50) of $h(\bar{t})$ are as follows (from divisions by $\bar{s}$)

$$\bar{H}_I(\bar{s}) = \frac{1}{\bar{s}}\left[\frac{K(1 - \exp(-\bar{s})) + K(\bar{s}(\lambda\vartheta)^{-1} + \bar{s}^2(\lambda\vartheta^2)^{-1}}{\bar{s}(\bar{s}^2(\lambda\vartheta^2)^{-1} + \bar{s}(\lambda\vartheta)^{-1} + 1)}\right] \tag{54}$$

$$\bar{H}_{II}(\bar{s}) = \frac{1}{\bar{s}}\left[\frac{\bar{s}K\left(1 + (\lambda\vartheta)^{-1}\right)\left(1 + \bar{s}(\lambda\vartheta)^{-1} + \bar{s}^2(\lambda\vartheta^2)^{-1}\right) - K\left(1 - e^{-\bar{s}} + \bar{s}(\lambda\vartheta)^{-1} + \bar{s}^2(\lambda\vartheta^2)^{-1}\right)}{\bar{s}^2\left(1 + \bar{s}(\lambda\vartheta)^{-1} + \bar{s}^2(\lambda\vartheta^2)^{-1}\right)}\right] \tag{55}$$

Finding the limits $h_I(\infty) = \lim_{\bar{s}\to 0} \bar{s}\bar{H}_I(\bar{s})$ and $h_{II}(\infty) = \lim_{\bar{s}\to 0} \bar{s}\bar{H}_{II}(\bar{s})$ we obtain

$$h_I(\infty) = \lim_{\bar{s}\to 0} \bar{s}\bar{H}_I(\bar{s}) = h(\infty)(1 + (\lambda\vartheta)^{-1}) \tag{56}$$

$$\lim_{\bar{s}\to 0} \bar{s}\bar{H}_{II}(\bar{s}) = K[0.5 + (\lambda\vartheta)^{-1} + (\lambda\vartheta)^{-2} - \lambda^{-1}\vartheta^{-2}] = h_{II}(\infty) \tag{57}$$

From the limits (56) and (57) we obtain the set of equations

$$\begin{aligned} &\frac{1}{\lambda\vartheta} = \frac{h_I(\infty)}{h(\infty)} - 1 = S_1 \\ &\frac{1}{\lambda\vartheta} + \frac{1}{(\lambda\vartheta)^2} - \frac{1}{\lambda\vartheta^2} = \frac{h_{II}(\infty)}{h(\infty)} - \frac{1}{2} = S_2 \end{aligned} \tag{58}$$

and the solution of this set is identical with (51)■.

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